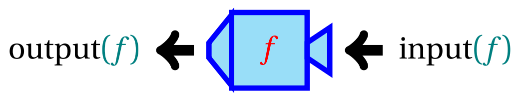
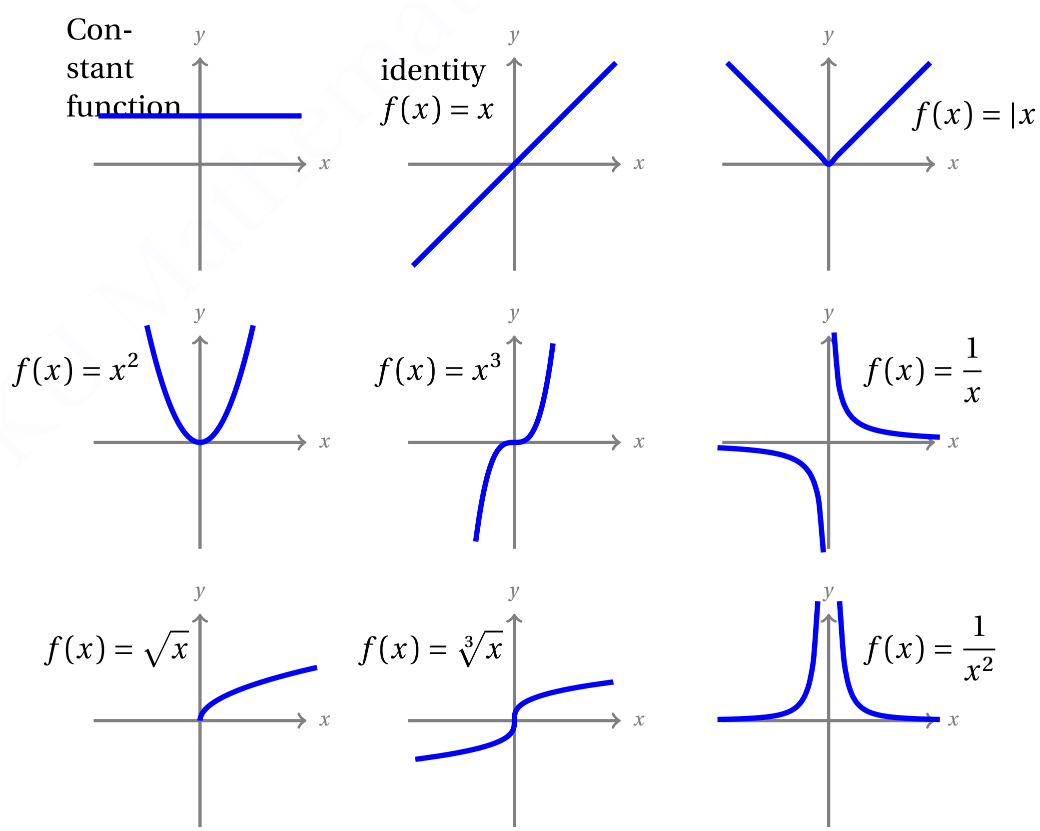
1.1: Functions

Definition and Notation of a Function



[1]

- A relation is a set of ordered pairs. A relation may be presented as a table or points on xy-plane.
- A function is a relation in which each input value results in **exactly one** output value. The presentation of function on xy-plane is called the graph of the function.
- A graph is a graph of a function if it satisfies the vertical line test. That is, if every vertical line only intersects the graph only once.
- A function is one-to-one if every output value is only associated with at most one input value. This can be tested with horizontal line test. That is, a function is one-to-one if every horizontal line intersects its graph in, at most, one point. (We learn about horizontal line test in Section 1.7.)
- Examples: The relation between letter grade and grade point average is a function.
- A few well-known graphs: (Memorize these and a few that we discuss in future chapters.) Make a table of sample points to graph. Use your arms in class to make a few of these shapes in the air.



[1] Note: In the figure for $y = f(x)$, the order of input and output, x on the right side of function and y on the left, is preserved.

Notation

- $f(x)$ reads f of x or f function of x. in $y = f(x)$ where x is the independent variable, or input, and y is the dependent variable, or output.

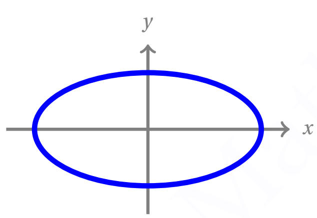
Domain and Range

- The set of all input values for function f is called the domain of f. (Denoted by D_f.)
- The set of all output values for function f is called the range of f. (Denoted by R_f.)
- We discuss domain and range in Section 1.2 and later on in the semester.

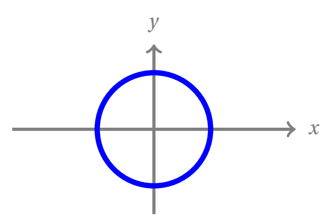
How to Evaluate Functions at a Value Using Their Rules[2]

- Identify the independent variable in the rule of function.
- Replace the independent variable with big parentheses.
- Plug in the input that needs to be evaluated inside the big parentheses.

1. Use vertical line test to identify the graphs of functions.



(A)



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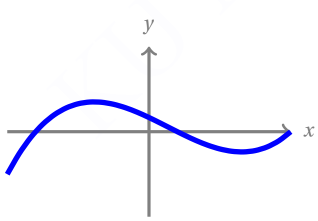

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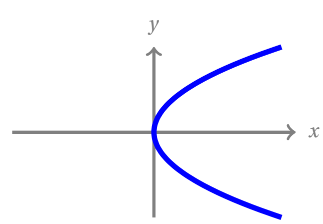

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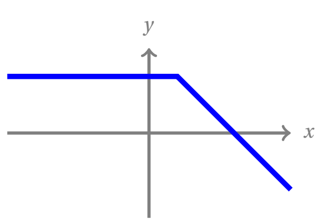

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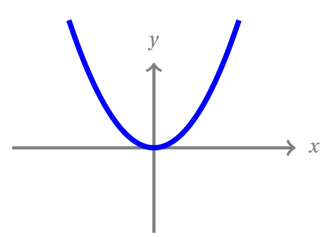


(F)

[2] The rule for a function $f(x)$ is a formula for f in terms of x.

Gateway Mastery-based Goals: Use the rule of the function to find the output for the given input.

2. Evaluate the function $f(x) = 3x^2 - 5x + 1$ for $x = -1$.

3. Evaluate the function $f(x) = 5x^2 - 10x$ for $x = b - 1$.

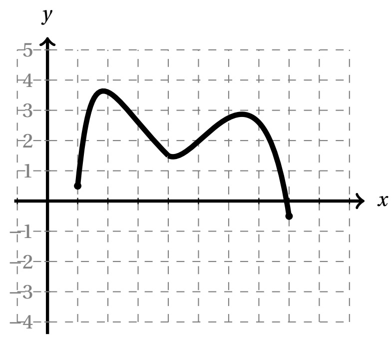
4. Evaluate the function $v(t) = 5t + 3$ for $t = a + h$.

5. Evaluate the function $f(y) = \dfrac{6y-1}{y}$ for $y = c + 2$.

6. Evaluate the function $h(s) = 3 - s - \dfrac{1}{2}s^2$ for $s = j - 2$.

7. The graph of the function $y = f(x)$ is shown to the right. For exactly how many values of x does $f(x) = 1$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
(F) 5



Related Videos

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Mini Projects: Choose one of the following mini projects that fits your interest best; seek others in your class who are in your major or are interested in the same project; complete the project with them in class; fill out the survey about the project on Canvas afterward.

8. **Biological Sciences, Mini Project 1:** (The velocity of blood in arteries)

As blood flows through a vein or an artery of a certain length, its velocity v is changing as the distance r from the central axis increases. The velocity v can be modeled as a function of r, for any artery with radius 0.6 mm, by the following formula:

$$v = 1900(0.36 - r^2) \qquad 0 \le r \le 0.6$$

Where r is in mm and v in mm/s

(a) Make a table of values of $v(r)$ for $r = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$ mm.

$r\,(mm)$	$v(r)\,(mm/s)$
0	
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	

(b) What does the table tell you about the change in the velocity of the flow of the blood in this artery? What is the maximum value of the velocity? What is the minimum value of the velocity?

(c) What is the domain of the function $v(r)$?

(d) What is the range of the function $v(r)$?

(e) The velocity v of blood in another artery of radius $R = 0.3$ mm is modeled by the function $v(r) = A(B - r^2)$ for constant A and B. What do you think B should be?

The arrows represent the velocity of the blood at different distances from the central axis in a cross section.

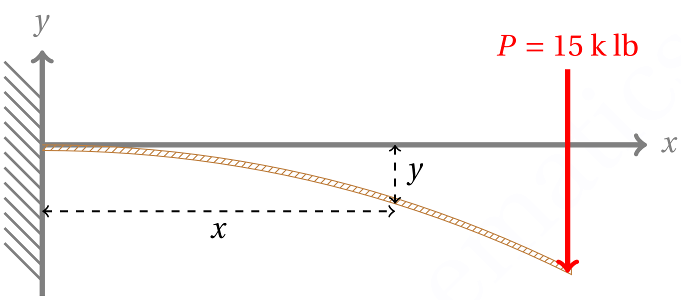
9. **Civil & Environmental Engineering, Mini Project 2:** (Single Load Cantilever Beam Deflection)

As a load is placed at the end of a simple supported beam at the other end, the vertical displacement of the beam from the horizontal, y in ft, at a horizontal distance $x\,ft$ from the support is best approximated by the function

$$y = \frac{-P}{EI}\left(0.5Lx^2 - \frac{x^3}{6}\right)$$

Where

x : The horizontal distance from the support in ft .
y : The vertical displacement in ft.
P : Point load in kilo pounds
L : The length of the beam in ft .
E : The Young's modulus, or modulus of elasticity, in kilo pounds/ft^2.
A property of materials that tells us how easily it can be deformed.
Materials that have a large E, such as metals, are difficult to deform.
Material that have small E, such as rubber, are easily deformed.
I : The moment of inertia in ft^4. This value is dependent on the beam's cross-section.
A metric for how the beam resists bending.
Beams with larger cross sections resist bending; if all the other conditions are the same for two beams with different cross sections, the thicker beam deflects less.



A $P = 15$ kilo pound load is placed at the free end of a simple supported beam of length $L = 20$ ft. The moment of inertia for the beam is $I = 0.0366\,ft^4$ and its Young modulus is $E = 4,175,000$ kilo pounds/ft^2.

(a) Use the numerical values of parameters E, I, L, and P to express y as a function of x only.

(b) Make a table of values of $y(x)$ for $x = 0, 5, 10, 15, 20$ ft.

$x(ft.)$	$y(ft.)$
0	
5	
10	
15	
20	

(c) What is the domain of the function $y(x)$?

(d) What is the maximum vertical deflection, $\delta = |y|$ and where does it occur?

(e) What is the range of $y(x)$?

(f) What are the roots (zeros) of the function in Part(A)?

10. **Aerospace Engineering, Mini Project 3:** (First-order Calculation of Expected Wing Deflection)

Aerospace engineers often approximate the wing as a simple cantilever beam that is fixed on one end (the fuselage[1] side) and free on the other.

The lift generated by the wings can be assumed to be distributed evenly across the span, b. The deflection of a wing can thus be calculated using the function: $y = \dfrac{\omega x^2}{24EI}(6L^2 - 4Lx + x^2)$

Where

x : The horizontal distance from the fuselage in inches.
y : The vertical deflection in inches.
b : The span of the wings in inches.
L : The length of each wing.($L = b/2$)
ω : The distributed lift load in pounds per inch.
E : The Young's modulus, or modulus of elasticity, in psi.
A property of materials that tells us how easily it can be deformed.
Materials that have a large E, such as metals, are difficult to deform.
Material that have small E, such as rubber, are easily deformed.
I : The moment of inertia. This value is dependent on the beam's cross-section.
A metric for how the beam resists bending.
Beams with larger cross sections resist bending; if all the other conditions are the same for two beams with different cross sections, the thicker beam deflects less.

The wing of a Twin Otter (next page), has a half-span, $L = b/2 = 390$ inches. During cruise, the distributed lift load is $\omega = 12.8\ lbs/in$. The moment of inertia of the wing is $I = 615\ in^4$. Finally, the wing is made of an aerospace grade aluminum whose modulus of elasticity is $E = 10^7$ psi.

(a) Use the numerical values of parameters E, I, L, and ω to express y as a function of x only.

(b) Make a table of values of $y(x)$ for $x = 0, 50, 100, 200, 300, 390$ in.

$x(in.)$	$y(in.)$
0	
50	
100	
200	
300	
390	

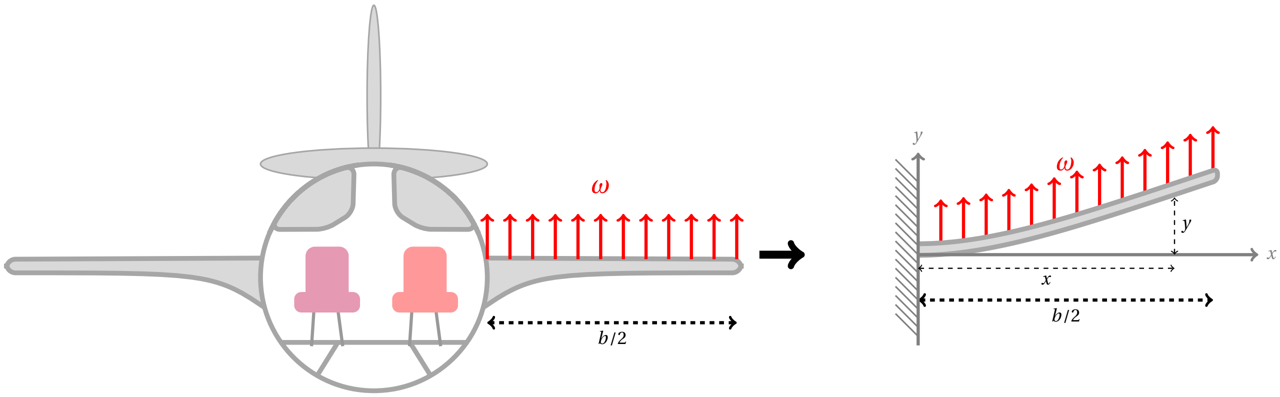
(c) What is the domain of the function $y(x)$?

(d) What is the maximum deflection of the wing and where does it occur?

(e) What is the range of $y(x)$?

(f) What are the roots (zeros) of the function in Part(A)?

[1]The main body of the aircraft.



The creation of engineering mini projects is an ongoing collaboration with the school of engineering. The Civil and Aerospace mini projects in this document were created in collaboration with professors Emily Arnold (Aerospace Engineering) and Bob Lyon (Civil and Environmental Engineering).

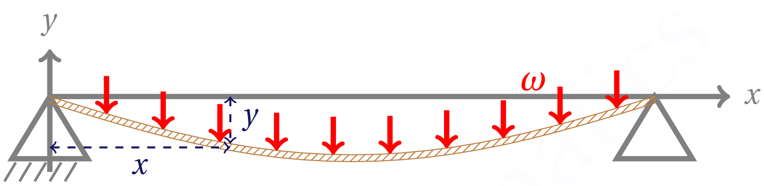
11. **Civil and Environmental Engineering, Mini Project 4:** (Deflection of a Simply supported bridge, Supported on both Ends, with a Uniformly Distributed Load)

As a load is distributed uniformly on a simple supported beam on both ends, the vertical displacement of the beam from the horizontal, y in ft, at a horizontal distance $x\,ft$ from the left support is best approximated by the function

$$y = \frac{-\omega x}{24EI}\left(L^3 - 2Lx^2 + x^3\right)$$

Where

x : The horizontal distance from the left support in ft .
y : The vertical displacement in ft .
ω : Load in kilo pounds
L : The length of the beam between two supports in ft .
E : The Young's modulus, or modulus of elasticity, in kilo pounds/ ft^2.
A property of materials that tells us how easily it can be deformed.
Materials that have a large E, such as metals, are difficult to deform.
Material that have small E, such as rubber, are easily deformed.
I : The moment of inertia in ft^4. This value is dependent on the beam's cross-section.
A metric for how the beam resists bending.
Beams with larger cross sections resist bending; if all the other conditions are the same for two beams with different cross sections, the thicker beam deflects less.



ω = 15 kilo pound load is uniformly distributed on a simply supported bridge, at ends. The length of the beam between two supports is $L = 20$ ft. The moment of inertia for the beam is $I = 0.0366\,ft^4$ and its Young modulus is $E = 4,175,000$ kilo pounds/ft^2.

(a) Use the numerical values of parameters E, I, L, and P to express y as a function of x only.

(b) Make a table of values of $y(x)$ for $x = 0, 5, 10, 15, 20$ ft.

$x(ft.)$	$y(ft.)$
0	
5	
10	
15	
20	

(c) What is the domain of the function $y(x)$?

(d) What is the maximum deflection, $\delta = |y|$, and where does it occur?

(e) What is the range of $y(x)$?

(f) What are roots (zeros) of the function in Part(A)?

12. **Mechanical Engineering, Mini Project 5:** (Fuel Efficiency of a vehicle within the City limits.)

As a car is traveling within the city speed limit, its fuel efficiency in mi/gal at speed $s\,mi/hr$ can be approximated by the function:

$$f(s) = -0.0035s^3 + 0.219s^2 - 3.675s + 39.829,$$

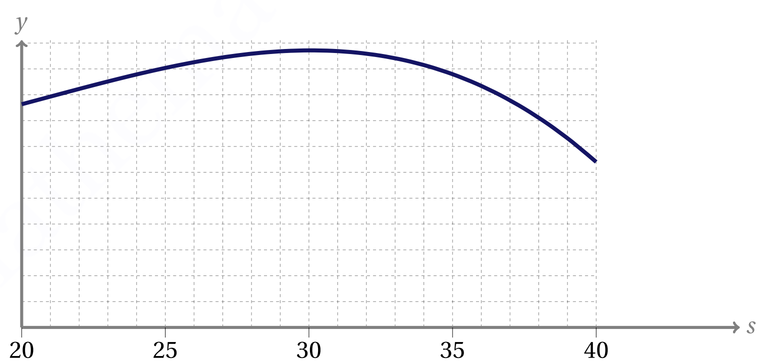
where s is the speed in mi/hr and $20 \le s \le 40$.

(a) What is the domain of $f(s)$?

(b) Find the fuel efficiency for the following speed values.

$s(mi/hr)$	Fuel efficiency $f(s)(mi/gal)$
20	
25	
30	
35	
40	

(c) The graph of the function $f(s)$ is shown below; what is the maximum fuel efficiency and at what speed is fuel efficiency maximum?



(d) What is the range of the function?

(e) A formula to approximate fuel efficiency of a second vehicle is given by

$$f(s) = As^3 + Bs^2 + Cs + D,$$

where $A = -0.0035$, $B = 0.219$, $C = -3.675$ and $D = 41.035$; is the second vehicle more efficient or less efficient that the first one? At what speed is this vehicle more fuel efficient?